Bayes Universal Source Coding Scheme for Correlated Sources

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Abstract—In this study, the problem of universal lossless codes is investigated. We consider Fixed-to-Variable length codes for i.i.d. correlated sources with one encoder and one decoder. Assuming a simple class of source models, we propose new coding scheme with Bayes codes. The asymptotic code length of the proposed codes for this class is also proved.

I. INTRODUCTION

The multiple correlated sources is defined as follows. We denote K data sequences with length \( n \) emitted from source by \( x^{(1)}(1), x^{(2)}(1), \cdots, x^{(K)}(1) \), where \( x^{(k)}(1) = x_{(1,1)}, x_{(1,2)}, \cdots, x_{(1,n)} \). For simplicity, we assume all \( K \) sequences are binary sequences. The probability of sequences is defined as follows:

\[
P(x^{(1)}(1), x^{(2)}(1), \cdots, x^{(K)}(1)) = \prod_{k=1}^{K} P(x^{(k)}(1), x^{(k)}(2), \cdots, x^{(k)}(n)) = \prod_{k=1}^{K} \sum_{\theta \in \Theta} P(x^{(k)}(1), x^{(k)}(2), \cdots, x^{(k)}(n) | \theta),
\]

where \( \theta \in \Theta \) is unknown parameter vector of distribution. All information sequences are encoded at once by one encoder. A receiver decodes all information sequences from code words. If the distribution of source sequences is given, there are some coding algorithm such as arithmetic coding algorithms that encodes sequences to code words whose mean code length converges to its entropy. Under the condition that such coding algorithm is used for coding, the decision of coding probability is the main problem in universal coding.

In this setting, one of the simplest ideas is to treat all \( K \) sequences as only one sequence of multiple alphabet that size is \( 2^K \). The simple Bayes coding scheme gives coding probability as follows:

\[
P_C(x^{(1)}(1), x^{(2)}(1), \cdots, x^{(K)}(1)) = \int_{\Theta} P(x^{(1)}(1), x^{(2)}(1), \cdots, x^{(K)}(1) | \theta) P(\theta) d\theta,
\]

where, \( P(\theta) \) is prior distribution of \( \theta \). The asymptotic mean code length of Bayes codes [1] is given by

\[
E \left[ -\log P_C(x^{(1)}(1), x^{(2)}(1), x^{(K)}(1)) \right] = n H(X(1), X(2), \cdots, X(K)) + \frac{2^K - 1}{2} \log n + O(1).
\]

II. PROPOSED SCHEME

The simultaneous distribution is given by

\[
P(x^{(1)}(1), x^{(2)}(1), \cdots, x^{(K)}(1) | \theta) = \prod_{k=1}^{K} P(x^{(k)}(1), x^{(k)}(2), \cdots, x^{(k)}(n) | \theta),
\]

where \( \lambda_k \) is some integer that satisfies \( \lambda_k \leq 2^{k-1} \). Therefore, the total mean code length is given by

\[
E \left[ -\log P_C(x^{(1)}(1), x^{(2)}(1), \cdots, x^{(K)}(1)) \right] = n H(X(1), X(2), \cdots, X(K)) + \frac{\lambda_k}{2} \log n + O(1).
\]

Since \( \sum_{k=1}^{K} \lambda_k \leq 2^K - 1 \), we have that our scheme is efficient than simple scheme.

REFERENCES