Bayes Universal Source Coding Scheme for Correlated Sources

Tota Suko, Shunsuke Horii, Toshiyasu Matsushima and Shigeichi Hirasawa
Waseda University
Tokyo, 169-8555 Japan.
Email: suko@aoni.waseda.jp

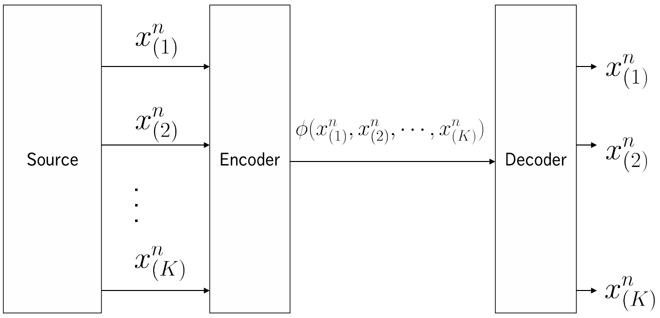

Fig. 1. system model

***Abstract*—In this study,the problem of universal lossless codes is investigated. We consider Fixed-to-Variable length codes for i.i.d. correlated sources with one encoder and one decoder. Assuming a simple class of source models, we propose new coding sheme with Bayes codes. The asymptotic code length of the proposed codes for this class is also proved.**

I. Introduction

The multiple correlated sources is defined as follows. We denote K data sequences with length n emitted from source by $x_{(1)}^n, x_{(2)}^n, \cdots x_{(K)}^n$, where $x_{(k)}^n = x_{(k),1}x_{(k),2}\cdots x_{(k),n}$. For simplicity, we assume all K sequences are binary sequences. The probability of sequences is defined as follows:

$$P(x_{(1)}^n, x_{(2)}^n, \cdots x_{(K)}^n|\theta) = \prod_{i=1}^n P(x_{(1),i}, x_{(2),i}, \cdots, x_{(K),i}|\theta), \quad (1)$$

where $\theta \in \Theta$ is unknown parameter vector of distribution. All information sequences are encoded at once by one encoder. A receiver decodes all information sequences from code words. If the distribution of source sequences is given, there are some coding algorithm such as arithmetic coding algorithms that encodes sequences to code words whose mean code length converges to its entropy. Under the condition that such coding algorithm is used for coding, the decision of coding probability is the main problem in universal coding.

In this setting, one of the simplest ideas is to treat all K sequences as only one sequence of multiple alphabet that size is 2^K. The simple Bayes coding scheme gives coding probability as follows:

$$P_C(x_{(1)}^n, x_{(2)}^n, \cdots x_{(K)}^n) = \int_\Theta P(x_{(1)}^n, x_{(2)}^n, \cdots x_{(K)}^n|\theta)P(\theta)d\theta, \quad (2)$$

where, $P(\theta)$ is prior distribution of θ. The asymptotic mean code length of Bayes codes [1] is given by

$$E\left[-\log P_C(x_{(1)}^n, x_{(2)}^n, \cdots x_{(K)}^n)\right] = nH(X_{(1)}, X_{(2)}, \cdots X_{(K)}) + \frac{2^K-1}{2}\log n + O(1). \quad (3)$$

II. Proposed scheme

The simultaneous distribution is given by

$$P(x_{(1),i}, x_{(2),i}, \cdots, x_{(K),i}|\theta) = P(x_{(1),i}|\theta_1)\prod_{k=2}^K P(x_{(k),i}|x_{(1),i}, \cdots, x_{(k-1),i}, \theta_k). \quad (4)$$

We propose coding scheme that sequences are coded individually. When we code $x_{(k)}^n$, we use $x_{(1)}^n, x_{(2)}^n, \cdots, x_{(k-1)}^n$ as side informations. And, the coding probability is given by

$$P_C^*(x_{(k)}^n|x_{(1)}^n, x_{(2)}^n, \cdots x_{(k-1)}^n) = \int_{\Theta_k} P(x_{(k)}^n|x_{(1)}^n, x_{(2)}^n, \cdots x_{(k-1)}^n|\theta_k)P(\theta_k)d\theta_k. \quad (5)$$

The asymptotic mean code length is given by

$$E\left[-\log P_C^*(x_{(k)}^n|x_{(1)}^n, x_{(2)}^n, \cdots x_{(k-1)}^n)\right] = nH(X_{(k)}|X_{(1)}, X_{(2)}, \cdots X_{(k-1)}) + \frac{\lambda_k}{2}\log n + O(1), (6)$$

where λ_k is some integer that satisfes $\lambda_k \leq 2^{k-1}$. Therefore, total mean code length is given by

$$E\left[-\sum_{k=1}^K \log P_C^*(x_{(k)}^n|x_{(1)}^n, x_{(2)}^n, \cdots x_{(k-1)}^n)\right] = nH(X_{(1)}, X_{(2)}, \cdots X_{(K)}) + \sum_{k=1}^K \frac{\lambda_k}{2}\log n + O(1). \quad (7)$$

Since $\sum_{k=1}^K \lambda_k \leq 2^K - 1$, we have that our scheme is efficient than simple scheme.

References

[1] B.S. Clark and A.R. Brron, “Information-theoretic asymptotics of Bayes methods,” IEEE Trans. Inf. Theory, vol.IT-36, no.3, pp.453–471, May 1990.
[2] T. Matsushima, H. Inazumi and S. Hirasawa, “A Class of Distortionless Codes Designed by Bayes Decision Theory,” IEEE Trans. Inf. Theory, vol.37, No.5, pp. 1288–1293, 1991.
[3] T. Matsushima and S. Hirasawa, “A Bayes coding algorithm for FSMX sources,” In Proc. Int. Symp. of Inf. Theory, pp.388, 1995.
[4] M. Gotoh, T. Matsushima, S.Hirasawa, “A generalization of B.S.Clarke and A. R. Barron’s asymptotics of Bayes codes for FSMX sources,” IEICE Trans. fundamentals, vol.E81-A, no.10, pp.2123–2132, 1998.